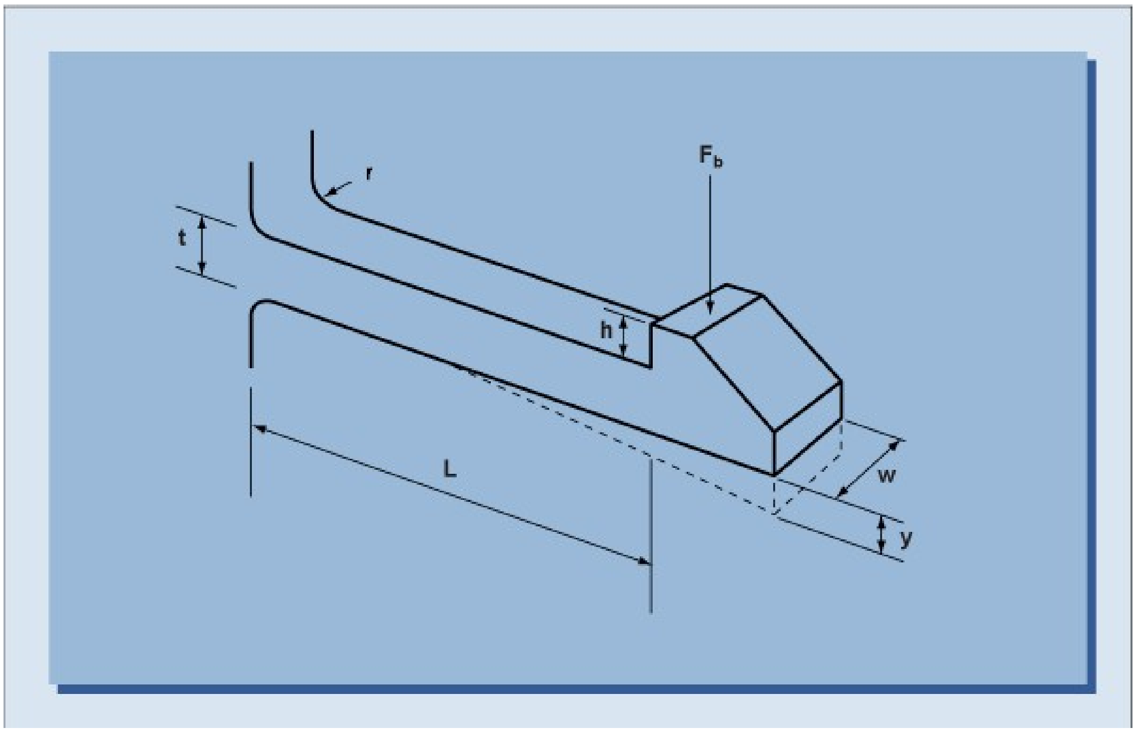
Snap fit design

Cantilever beam snap-fits

Cantilever beam type snap-fits can be calculated using a simplification of the general beam theory. In general, the stiffness of the part to which the snap-fit connects, is important. The formulae mentioned only roughly describe the behavior of both the part geometry and the material. On the other hand, the approach can be used as a first indication if a snap-fit design and material choice are feasible.

Cantilever beam with constant rectangular cross section

A simple type of snap-fit, the cantilever beam, is demonstrated in the figure below, which shows the major geometrical parameters of this type of snap-fit. The cross section is rectangular and is constant over the whole length L of the beam.



The maximum allowable deflection y and deflection force F_b can be calculated with the following formulas if the maximum allowable strain level ε of the material is known.

$$y = \frac{2}{3} \cdot \frac{L^2}{t} \cdot \varepsilon$$

$$F_b = \frac{w \cdot t^2 \cdot E_s}{6 \cdot L} \cdot \varepsilon$$

where

E_s = secant modulus
L = length of the beam
t = height of the beam
w = width of the beam
ε = maximum allowable strain level of the material

The four dimensions that can be changed by the designer are:

- **h**, the height of the snap-fit lip. Changing the height might reduce the ability of the snap-fit to ensure a proper connection.
- **t**, the thickness of the beam. A more effective method is to use a tapered beam. The stresses are more evenly spread over the length of the beam.
- increasing the length of beam, **L**, is the best way to reduce strain as it is represented squared in the equation for the allowable deflection.
- the deflection force is proportional to the width, **w**, of the snap-fit lip.

Beams with other cross sections

The following general formulae for the maximum allowable deflection y and deflection force F_b can be used for cantilever beams with a constant asymmetric cross section.

$$y = \frac{L^2}{3 \cdot e} \cdot \varepsilon$$

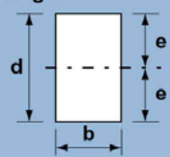
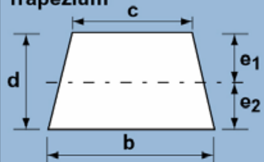
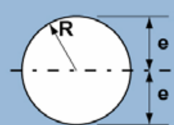
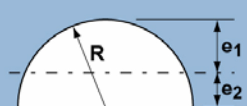
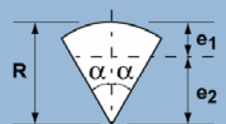
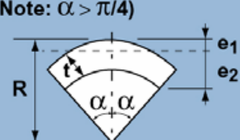
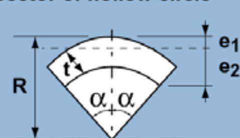
$$F_b = \frac{E_s \cdot I}{e \cdot L} \cdot \varepsilon$$

where

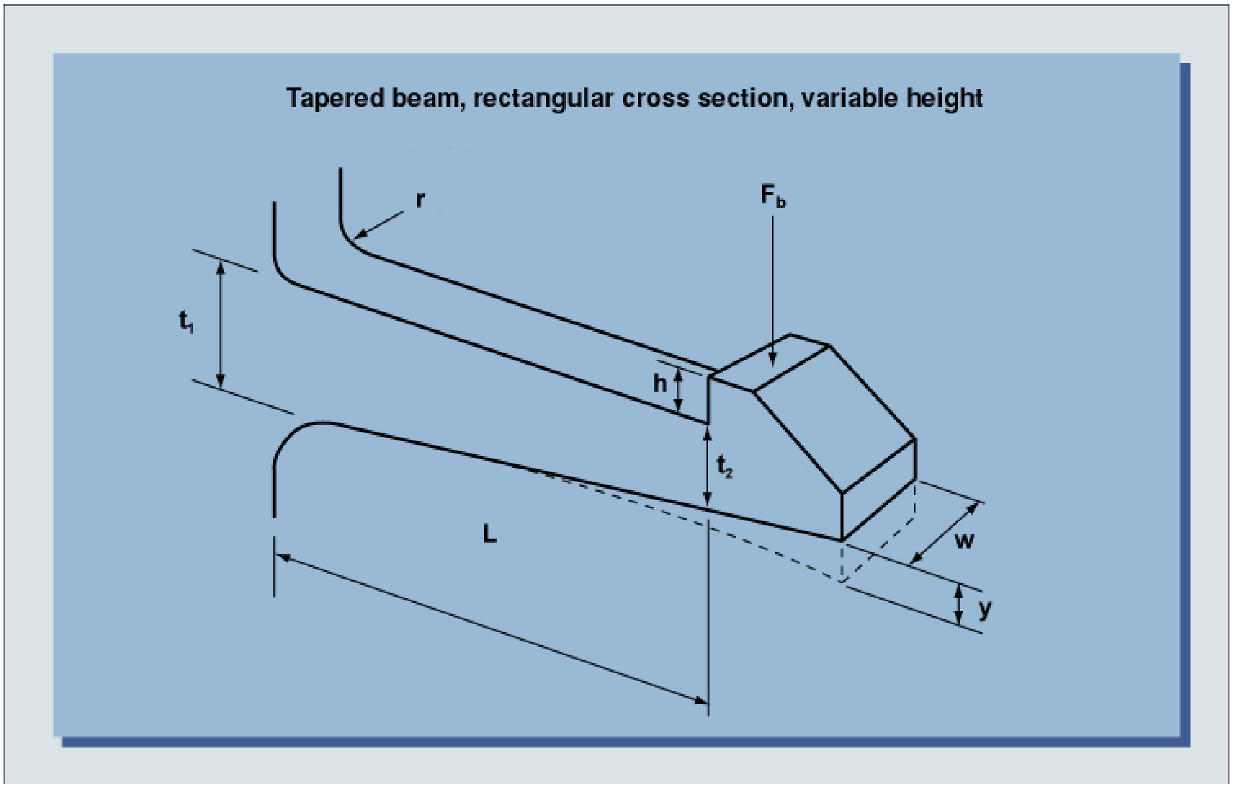
E_s = secant modulus
I = moment of inertia of the cross section
L = length of the beam
e = distance from the neutral line in the cross section to the extreme fibre
ε = maximum allowable strain level of the material

Normally tensile stresses are more critical than compressive stresses. Therefore the extreme fiber distance **e** that belongs to the side under tension is used in the above-mentioned formulae.
The moment of inertia and the extreme fiber distance is given in table 1 for some cross sections.

Moment of inertia and extreme fibre distance

Form of section	Area and distance from centroid to extremities	Moment of inertia
Rectangle 	$A = b \cdot d$ $e = \frac{d}{2}$	$I = \frac{1}{12} \cdot b \cdot d^3$
Trapezium 	$A = d \cdot (b + c) / 2$ $e_1 = \frac{d}{3} \cdot \frac{2b + c}{b + c}$ $e_2 = d - e_1$	$I = \frac{d^3}{36} \cdot \frac{b^2 + 4 \cdot b \cdot c + c^2}{b+c}$
Solid circle 	$A = \pi \cdot R^2$ $e = R$	$I = \frac{\pi}{4} \cdot R^4$
Solid semicircle 	$A = \frac{\pi}{2} \cdot R^2$ $e_1 = 0.5756 \cdot R$ $e_2 = 0.4244 \cdot R$	$I = 0.1098 \cdot R^4$
Sector of solid circle 	$A = \alpha \cdot R^2$ $e_1 = R \cdot \left(1 - \frac{2 \cdot \sin\alpha}{3 \cdot \alpha}\right)$ $e_2 = \frac{2 \cdot R \cdot \sin\alpha}{3 \cdot \alpha}$	$I = \frac{R^4}{4} \left(\alpha + \sin\alpha \cdot \cos\alpha - \frac{16 \cdot \sin^2\alpha}{9 \cdot \alpha}\right)$
Segment of solid circle (Note: $\alpha > \pi/4$) 	$A = R^2 \cdot (\alpha - \sin\alpha \cdot \cos\alpha)$ $e_1 = R \cdot \left[1 - \frac{2 \cdot \sin^3\alpha}{3 \cdot (\alpha - \sin\alpha \cdot \cos\alpha)}\right]$ $e_2 = R \cdot \left[\frac{2 \cdot \sin^3\alpha}{3 \cdot (\alpha - \sin\alpha \cdot \cos\alpha)} - \cos\alpha\right]$	$I = \frac{R^4}{4} \cdot [\alpha - \sin\alpha \cdot \cos\alpha +$ $2 \cdot \sin^3\alpha \cdot \cos\alpha - \frac{16 \cdot \sin^6\alpha}{9 \cdot (\alpha - \sin\alpha \cdot \cos\alpha)}]$
Sector of hollow circle 	$A = \alpha \cdot t \cdot (2 \cdot R - t)$ $e_1 = R \cdot \left[1 - \frac{2 \cdot \sin\alpha}{3 \cdot \alpha} \cdot \left(1 - \frac{t}{R} + \frac{1}{2 - t/R}\right)\right]$ $e_2 = R \cdot [\frac{2 \cdot \sin\alpha}{3 \cdot \alpha \cdot (2 - t/R)}$ $+ \left(1 - \frac{t}{R}\right) \cdot \frac{2 \cdot \sin\alpha - 3 \cdot \alpha \cdot \cos\alpha}{3 \cdot \alpha}]$	$I = R^3 \cdot t \cdot [(1 - \frac{3 \cdot t}{2 \cdot R} + \frac{t^2}{R^2} - \frac{t^3}{4 \cdot R^3})$ $\times \left(\alpha + \sin\alpha \cdot \cos\alpha - \frac{2 \cdot \sin^2\alpha}{\alpha}\right)$ $+ \frac{t^2 \cdot \sin^2\alpha}{3 \cdot R^2 \cdot \alpha \cdot (2 - t/R)} \cdot \left(1 - \frac{t}{R} + \frac{t^2}{6 \cdot R^2}\right)]$

Tapered beams with a variable height



The following formulae can be used to calculate the maximum allowable deflection y and deflection force F_b for a tapered cantilever beam with a rectangular cross section. The height of the cross section decreases linearly from t_1 to t_2, see figure above.

$$\mathbf{y} = c \cdot \frac{2 \cdot L^2}{3 \cdot t_1} \cdot \varepsilon$$

$$F_b = \frac{w \cdot t_1^2 \cdot E_s}{6 \cdot L} \cdot \varepsilon$$

where

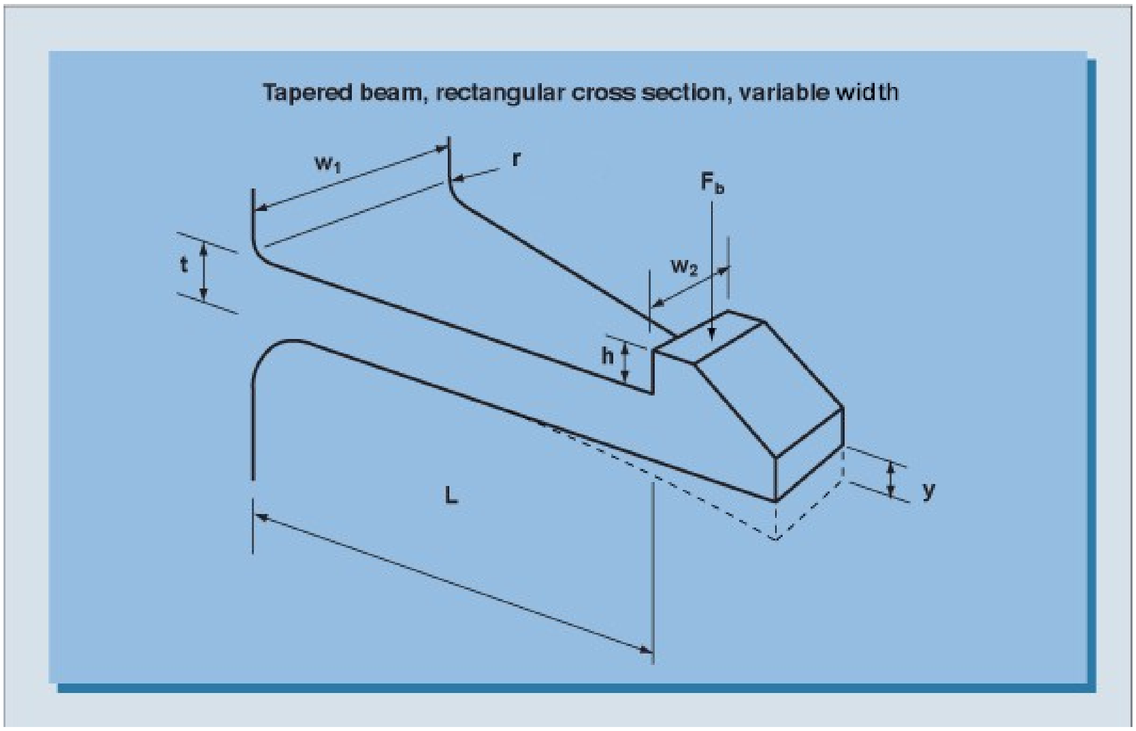
E_s = secant modulus
L = length of the beam
c = multiplier
w = width of the beam
$\mathbf{t_1}$ = height of the cross section at the fixed end of the beam
ε = maximum allowable strain level of the material

The formula for the deflection **y** contains a multiplier **c** that depends on the ratio $\mathbf{t_2 / t_1}$, see table 2, where t_1 is the height of the beam at the fixed end and t_2 is the height of the beam at the free end.

Table 2. Multiplier c as a function of the height

t_2 / t_1	0.40	0.50	0.60	0.70	0.80	0.90	1.00
c	1.893	1.636	1.445	1.297	1.179	1.082	1.000

Tapered beams with a variable width



The following formulae can be used to calculate the maximum allowable deflection **y** and deflection force $\mathbf{F_b}$ for a tapered cantilever beam with a rectangular cross section. The width of the cross section decreases linearly from w_1 to w_2, see figure above.

$$\mathbf{y} = c \cdot \frac{2 \cdot L^2}{3 \cdot t} \cdot \varepsilon$$

$$\mathbf{F_b} = \frac{w_1 \cdot t^2 \cdot E_s}{6 \cdot L} \cdot \varepsilon$$

where

E_s = secant modulus

L = length of the beam
c = multiplier
$\mathbf{w_1}$ = width of the beam at the fixed end of the beam
t = height of the cross section
ε = maximum allowable strain level of the material

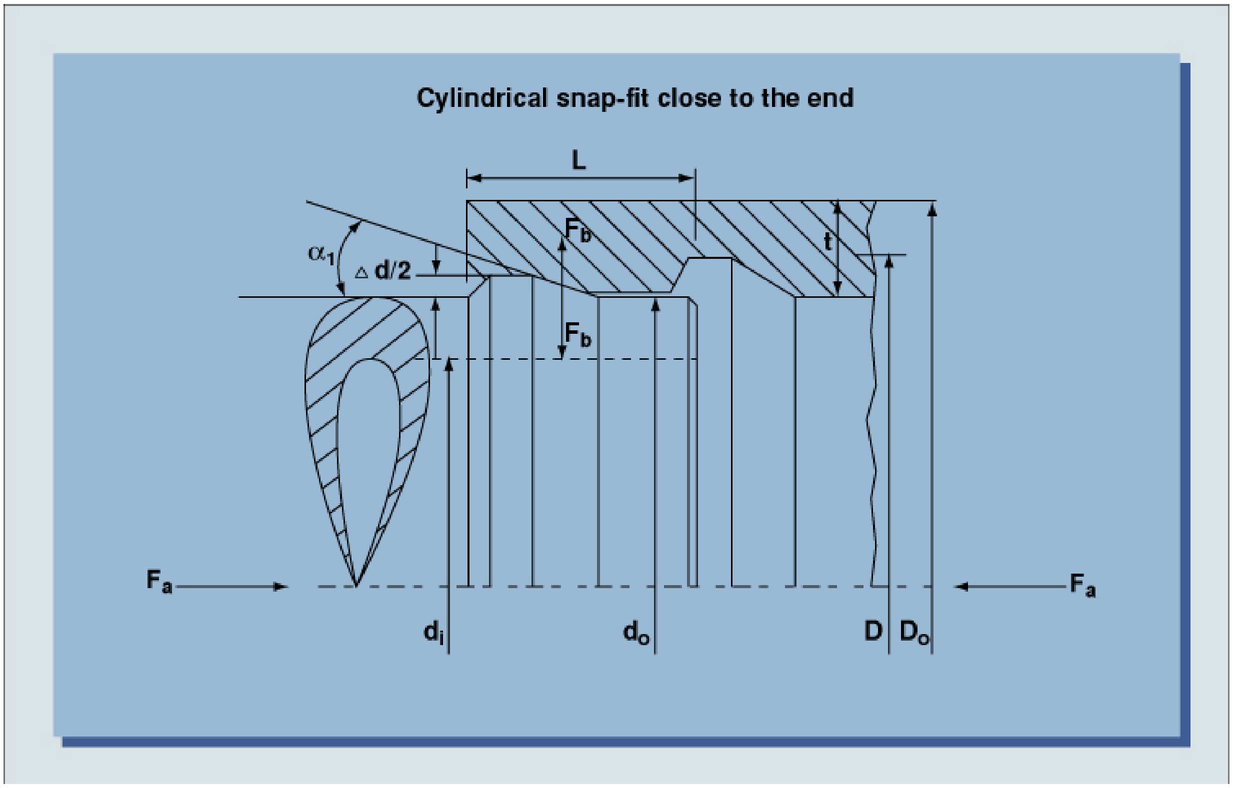
The multiplier **c** depends on the ratio $\mathbf{w_2}$ / $\mathbf{w_1}$, see table 3, where w_1 is the width of the beam at the fixed end and w_2 is the width of the beam at the free end.

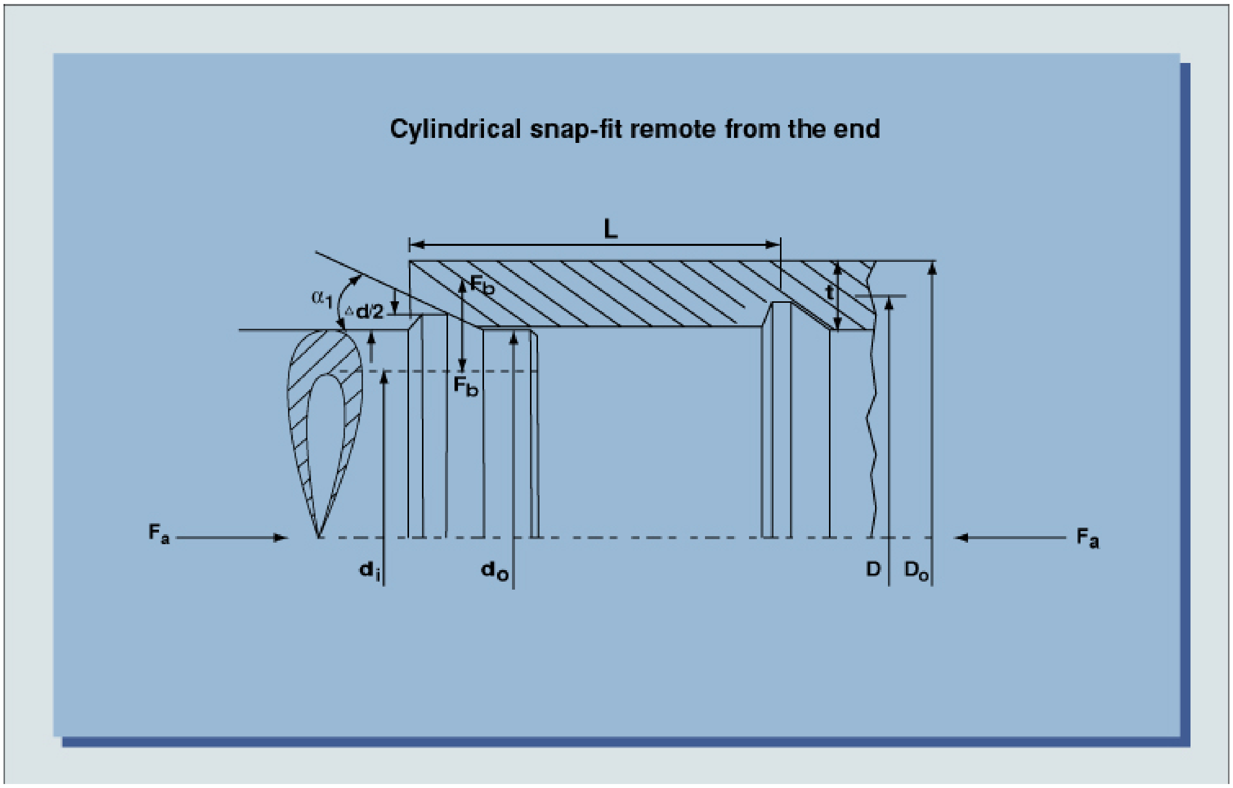
Table 3. Multiplier c as a function of the width

w_2 / w_1	0.125	0.25	0.50	1.00
c	1.368	1.284	1.158	1.000

Cylindrical snap-fits

One must distinguish between a cylindrical snap-fit close to the end of the pipe or remote from the end





More material must be deformed if the snap fit is remote from the end, and the deflection force F_b and mating force F_a will be a factor 3.4 higher. The snap-fit is regarded as being remote if

l > 1.8 . √ (D . t)

where **l** = distance to the end of the pipe.

The following symbols are further used:

D = average diameter of the pipe = (D_o + d_o) / 2
D_o = outside diameter of the pipe
d_o = outside diameter of the shaft
d_i = inside diameter of the shaft
Δd / 2 = height of the bulge on the shaft = depth of the groove in the pipe
E_s = shear modulus of the plastic
t = wall thickness of the pipe = (D_o – d_o) / 2
μ = coefficient of friction
v = Poisson's ratio of the plastic
The formula for the deflection force F_b is given in table 4 for a rigid (metal) shaft and a flexible pipe, respectively a flexible shaft and a rigid (metal) pipe. Four cases can be distinguished.

Table 4. Deflection force F_b		
Snap-fit close to the end	Rigid shaft, flexible pipe	$0.62 \cdot \Delta d \cdot d_o \cdot \dfrac{\sqrt{(D_o / d_o - 1) / (D_o / d_o + 1)}}{[(D_o / d_o)^2 + 1] / [(D_o / d_o)^2 - 1] + v} \cdot E_s$
	Flexible shaft, rigid pipe	$0.62 \cdot \Delta d \cdot d_o \cdot \dfrac{\sqrt{(d_o / d_i - 1) / (d_o / d_i + 1)}}{[(d_o / d_i)^2 + 1] / [(d_o / d_i)^2 - 1] - v} \cdot E_s$
Snap-fit remote from the end	Rigid shaft, flexible pipe	$2.1 \cdot \Delta d \cdot d_o \cdot \dfrac{\sqrt{(D_o / d_o - 1) / (D_o / d_o + 1)}}{[(D_o / d_o)^2 + 1] / [(D_o / d_o)^2 - 1] + v} \cdot E_s$
	Flexible shaft, rigid pipe	$2.1 \cdot \Delta d \cdot d_o \cdot \dfrac{\sqrt{(d_o / d_i - 1) / (d_o / d_i + 1)}}{[(d_o / d_i)^2 + 1] / [(d_o / d_i)^2 - 1] - v} \cdot E_s$

If the deflection force F_b according table 4 has been calculated, the mating force F_a is found using the expression

$$F_a = F_b \cdot \frac{\mu + \tan \alpha_1}{1 - \mu \cdot \tan \alpha_1}$$

The highest tangential strain ε_φ in the plastic is approximately:

- Rigid shaft, flexible pipe: $\varepsilon_\varphi = \Delta d / d_o$ (tension in the pipe)
- Flexible shaft, rigid pipe: $\varepsilon_\varphi = - \Delta d / d_o$ (compression in the shaft)

The highest axial bending strain ε_a in the plastic is about a factor 1.59 higher :

$\varepsilon_a = 1.59 \cdot \varepsilon_\varphi$ (tension at one side and compression at the other side)

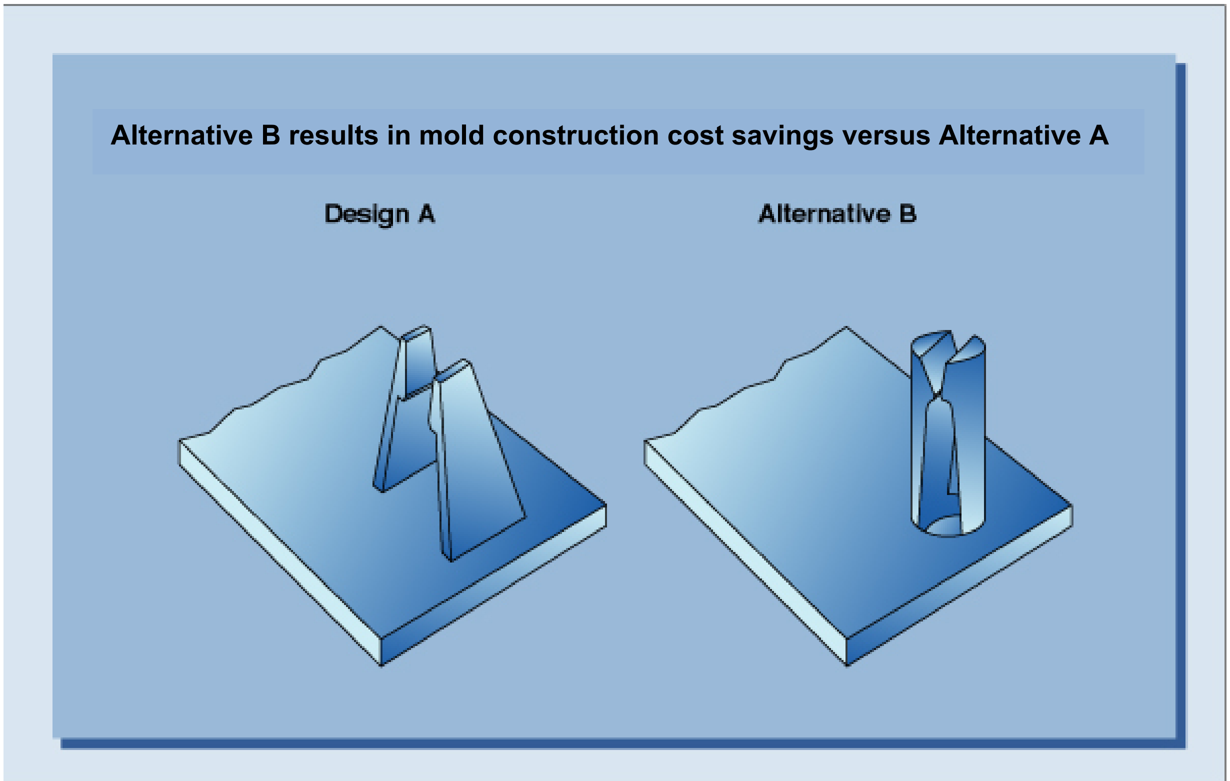
The calculation procedure when both parts are flexible and both are deformed is explained in the theory of snap fits. As a first approach, for flexible materials with a comparable stiffness, one can assume that the total deformation Δd is equally divided between the two parts.

Spherical snap-fits

The spherical snap-fit can be regarded as a special case of the cylindrical snap-fit. The formulas for a cylindrical snap-fit close to the end of the pipe can be used.

Mold construction

Mold construction costs are highly affected by the design of the snap fit.



For design A, an expensive slide in the mold is required and the flat surfaces require expensive milling. No slide is required for alternative B and the cylindrical outside surface can simply be drilled.